

Completely Randomized Design (CRD)

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Objective:

To Analyze Completely Randomized Design (CRD) with equal replications

The following table gives the yields (in kg per plot) of a vegetable using four treatments — A-Organic, B-Chemical, C-Mixed, and D-Control applied to each of four plots in a Completely Randomized Design (CRD). Test whether the treatments have the same effect on yield.

Yield (in kg per plot) of vegetable						
Treatment	Replications (R_j)				Total	Mean (Round figure)
A-Organic	50	55	52	58	$T_A = 215$	$\bar{Y}_A = 54$
B-Chemical	60	62	65	58	$T_B = 245$	$\bar{Y}_B = 61$
C-Mixed	70	75	72	68	$T_C = 285$	$\bar{Y}_C = 71$
D-Control	45	48	50	47	$T_D = 190$	$\bar{Y}_D = 48$
Grand Total (GT)					935	

Solution:

Linear model

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} ; \quad (i=1,2, \dots, k; j=1,2, \dots, r)$$

where Y_{ij} = Response of the j^{th} replication and i^{th} treatment

μ = General mean effect

α_i = i^{th} treatment effect

ε_{ij} = error effect with mean=0 and variance $=\sigma^2$ [$N(0, \sigma^2)$]

Hypothesis

H_0 : All treatments have equal effect on yield ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$)

H_1 : All α_i 's are not equal

Here No. of Treatments (k) = 4, No. of Replications (r) = 4,

No. of Observations (n) = $k \times r = 16$

$$\text{Correction Factor (C.F.)} = \frac{GT^2}{n} = \frac{(935)^2}{16} = 54639.06$$

$$\begin{aligned}\text{Total Sum of Square (TSS)} &= \sum_{i=1}^k \sum_{j=1}^r y_{ij}^2 - C.F. \\ &= (50^2 + 55^2 + \dots + 50^2 + 47^2) - 54639.06 = 1357.94\end{aligned}$$

$$\begin{aligned}\text{Treatment Sum of Squares (SST)} &= \frac{\sum_{i=1}^k T_i^2}{r} - C.F. \\ &= \frac{(T_A^2 + T_B^2 + T_C^2 + T_D^2 + T_E^2)}{r} - C.F. \\ &= \frac{(215^2 + 245^2 + 202^2 + 190^2)}{4} - 54639.06 \\ &= 1254.69\end{aligned}$$

$$\text{Error Sum of Squares (SSE)} = \text{T.S.S.} - \text{S.S.T.} = 1357.97 - 1254.69 = 103.25$$

ANOVA table

Source of variation	Degree of Freedom	Sum of Squares	Mean Sum of Squares	F _{cal}	F _(3,12)	
					5%	1%
Treatments	k-1=3	1254.69	418.23	48.61**	3.49	5.95
Error	n-k=12	103.25	8.60			
Total	n-1=15					

Since $F_{cal}=48.61 > F_{tab}=5.95$, Therefore null hypothesis H_0 at (1%) level of significance with degree of freedom (3,12) will be rejected. Hence there is a highly significant difference between the treatments.

To test the treatments (pair wise) arranges means in descending order

$$\bar{Y}_C=71, \bar{Y}_B=61, \bar{Y}_A=54, \bar{Y}_D=48$$

Standard error of difference between two treatments

$$SE(d) = \sqrt{EMS \left(\frac{1}{r_i} + \frac{1}{r_j} \right)} = \sqrt{Ve \left(\frac{1}{r_i} + \frac{1}{r_j} \right)} = \sqrt{8.6 \left(\frac{1}{4} + \frac{1}{4} \right)} = 2.07$$

$$\begin{aligned}\text{Critical difference (C.D.)} &= SE(d) \times t_{\text{error d.f.}} (5\%) = 2.07 \times t_{12} (5\%) \\ &= 2.07 \times 2.18 = 4.52\end{aligned}$$

Varieties	Difference of treatments mean	Compare with CD value
C-B	10	Significant
C-A	18	Significant
C-D	24	Significant
B-A	8	Significant
B-D	14	Significant
A-D	6	Significant

$$\text{Grand total} = 215 + 245 + 285 + 190 = 935$$

$$\text{Grand Mean} = \frac{GT}{n} = \frac{935}{16} = 58.44$$

$$CV = \frac{\sqrt{MSE}}{\bar{y}} \times 100 = \frac{\sqrt{8.6}}{58.44} \times 100 = 5.01\%$$

Conclusion: -

Treatment C gives significantly higher yield than all other treatments. So, C is found to be superior one among four treatments.

Exercise-1:

To find out the yielding ability of five varieties of sesamum an experiment was conducted in the greenhouse using a CRD with four plots per variety. The results are given in the table below.

Seed yield of sesamum g./plot

V ₁	V ₂	V ₃	V ₄	V ₅
25	25	24	20	14
21	28	24	17	15
21	24	16	16	13
18	25	21	19	11

Exercise-2

Objective:

To Analyze Completely Randomized Design (CRD) for unequal replications

A Completely Randomized Design was conducted with the three treatments A, B and C where treatment A is replicated 6 times and B is replicated 4 times and C is replicated 5 times. Analyze the data and state your conclusions.

	R ₁	R ₂	R ₃	R ₄	R ₅	R ₆
A	16.5	17	16	13	18	14
B	15	13.8	14	10.9		
C	18.2	24.2	25	28.9	21	